Model studies of the frequency-dependent attenuation of vibrations by viscoelastic polymers

S. Yazdani-Ardakani
Corporate Research, Cardinal Industries, Columbus, OH 43068, USA

and S. K. Kesavan
Department of Chemical Engineering, The University of Akron, Akron, OH 44325, USA
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An experimental technique designed to assess the frequency-dependent shock-absorption characteristics of viscoelastic polymers has been developed. This technique uses a vibration exciter to subject a loaded polymer specimen to vibrations similar to that expected under conditions of actual application. Accelerometers are used to monitor the attenuation of the input vibrations. The antivibration characteristics of the tested polymer are quantitatively represented by means of an amplitude ratio. Results obtained for a viscoelastic polymer have been presented to illustrate the application of the testing procedure.

(Keywords: vibration testing; viscoelastic polymers; shock-absorbing polymers; antivibration polymers; vibration attenuation; polymer testing; transfer function; frequency-dependent vibration attenuation)

INTRODUCTION

A variety of viscoelastic polymers are used in industrial applications to attenuate the transmission of vibrations to mechanical structures. Shock-absorbing polymers are also used in biomechanical applications to reduce the impulse wave transmission to the lower skeletal structure during locomotion[1].

Apart from material characteristics, the shock-absorbing properties of viscoelastic polymers depend on material thickness and both the waveform and frequency of the imposed oscillations. For a particular application, each polymer has a frequency range over which it is ineffective as a shock absorber because of resonance—a situation in which the amplitude of the transmitted force with respect to the input force would theoretically tend to infinity. Hence, a knowledge of the frequency dependence of the antivibration characteristics of a candidate polymer is necessary to determine its suitability for a particular application.

In this paper, we have presented results obtained from model studies designed to assess the shock-absorbing characteristics of polymers as a function of driving frequency. This technique uses a vibration exciter to simulate oscillating input force. The vibrations are transmitted through a specimen of the polymer to be tested. The polymer is loaded with a weight to simulate loading conditions expected in actual application. The attenuation of the vibrations gives a measure of the shock-absorbing characteristics of the polymer. By varying the frequency of vibration, the frequency dependence of shock absorption was tested.

THEORY

Our vibrating system consisting of a loaded viscoelastic shock-absorbing polymer is mathematically modelled as a system with ground excitation and a single degree of freedom (*Figure 1*). The governing differential equation for this system is:

$$m\ddot{x}+C(\dot{x}-\dot{y})+K(x-y)=0 \qquad (1)$$

where M is the mass, C is the viscous damping coefficient, K is the spring constant, x is the system response and y is the ground excitation.

We next define a quantity $z=x-y$. Using this definition for z in equation (1), we obtain:

$$M\ddot{z}+C\dot{z}+Kz=-M\ddot{y} \qquad (1a)$$

For a periodic signal, y can be defined as:

$$y=Ae^{j\omega t}$$

Differentiating this expression for y, we obtain the first and second differentials as:

$$\dot{y}=Aj\omega e^{j\omega t} \qquad \ddot{y}=-A\omega^2 e^{j\omega t}$$

Using the equation for $\ddot{y}$ in equation (1a) and solving for z, we obtain:

$$z=\frac{AM\omega^2}{[(K-M\omega^2)^2+(C\omega)^2]^{1/2}}e^{(j\omega t-\phi)} \qquad (2)$$

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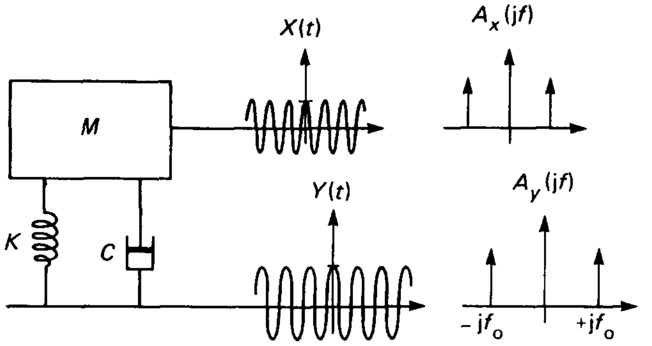

Figure 1 Free-body diagram of the vibrating system

where ϕ is the phase angle given by

$$\phi = \tan^{-1}\left(\frac{C\omega}{K - M\omega^2}\right)$$

The ratio x/y (or $(z+y)/y$) is generally defined[2] as the transfer function $H(j\omega)$. We next define a quantity D as:

$$D = \frac{M\omega^2}{[(K - M\omega^2)^2 + (C\omega)^2]^{1/2}}$$

Hence we can write an expression for $H(j\omega)$ as

$$H(j\omega) = \frac{AD\mathrm{e}^{(j\omega t - \phi)} + A\mathrm{e}^{j\omega t}}{A\mathrm{e}^{j\omega t}} = 1 + D\mathrm{e}^{-\phi} \quad (3)$$

Some mathematical manipulation of this expression yields:

$$H(j\omega) = \frac{K^2 - KM\omega^2 - jCM\omega^3 + (\omega C)^2}{(K - M\omega^2)^2 + (C\omega)^2} \quad (4)$$

If we determine the transfer function from our experiments with the viscoelastic polymer, we will succeed in quantifying the shock-absorption characteristics of the polymer. However, the calculation of $H(j\omega)$ is not possible for our system as we have no knowledge of C and K in equation (4). Therefore, by performing experiments and using Fourier series[3], the transfer function has been determined. The Fourier series is given by:

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t) \quad (5)$$

where

$$a_n = \frac{\omega}{n} \int_{-\pi/\omega}^{\pi/\omega} x(t) \cos n\omega t \, \mathrm{d}t$$

$$b_n = \frac{\omega}{n} \int_{-\pi/\omega}^{\pi/\omega} x(t) \sin n\omega t \, \mathrm{d}t$$

$$\omega = 2\pi f$$

and f is frequency (in Hz).

Manipulation of equation (5) yields the Fourier transform pair given by:

$$A_x(jf) = \int_{-\infty}^{\infty} x(t)\mathrm{e}^{-j2\pi f t}\mathrm{d}t \quad (6a)$$

$$x(t) = \int_{-\infty}^{\infty} A_x(jf)\mathrm{e}^{j2\pi f t}\mathrm{d}f \quad (6b)$$

Equation (5) can be used to represent only the periodic signals, whereas equations (6a) and (6b) can be used to define any non-periodic signal. The Fourier analyser used in our experiments (described below) uses equation (6a) to perform fast Fourier transformation.

EXPERIMENTAL APPARATUS AND PROCEDURE

A schematic diagram of the experimental set-up is shown in *Figure 2*. A disc-like specimen (3.0 inch diameter) of the polymer to be tested was placed in the mild steel retaining cup. Composite specimens consisting of a number of stacked discs can also be tested. A commercially available viscoelastic polymeric material (trade name 'Sorbothane') was tested for frequency dependence of shock-absorption characteristics. The polymer was loaded with the required weight to simulate actual loading conditions. Miniature Bruel and Kjaer piezoelectric accelerometers were mounted on the base of the cup and on the weight to monitor the input and output oscillations (*Figure 2*).

A Bruel and Kjaer (model 4801) vibration exciter was used to simulate the vibrations expected under conditions of actual application (*Figure 2*). An impulsive waveform provided by a sinusoidal signal sweep generator (Bruel and Kjaer, type 1024) was used to drive the shaker with adjustable amplitude of vibration. The signal sweep generator was capable of generating and sweeping signals

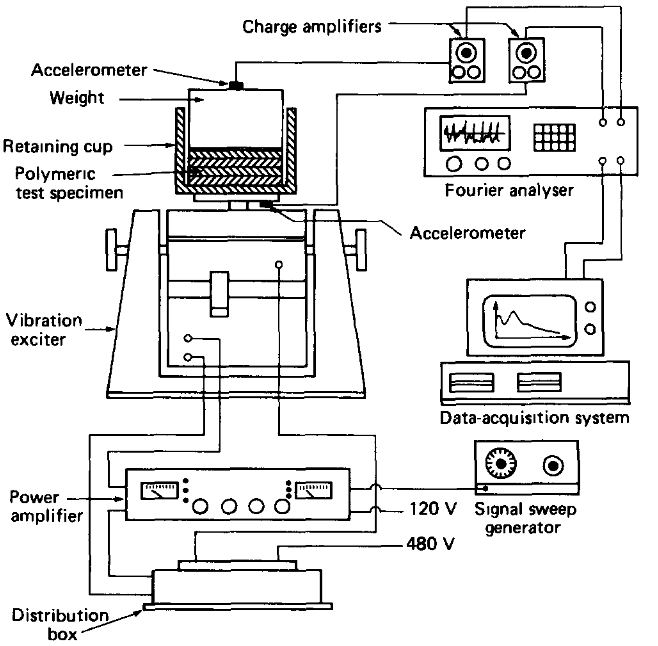

Figure 2 Experimental set-up

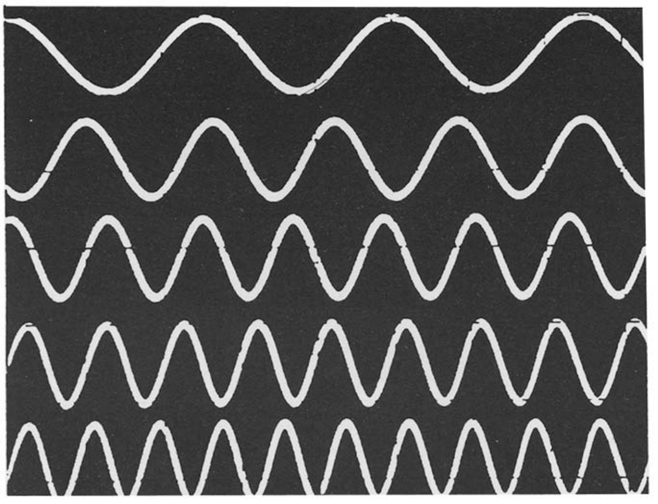

Figure 3 Typical input oscillations provided by the exciter

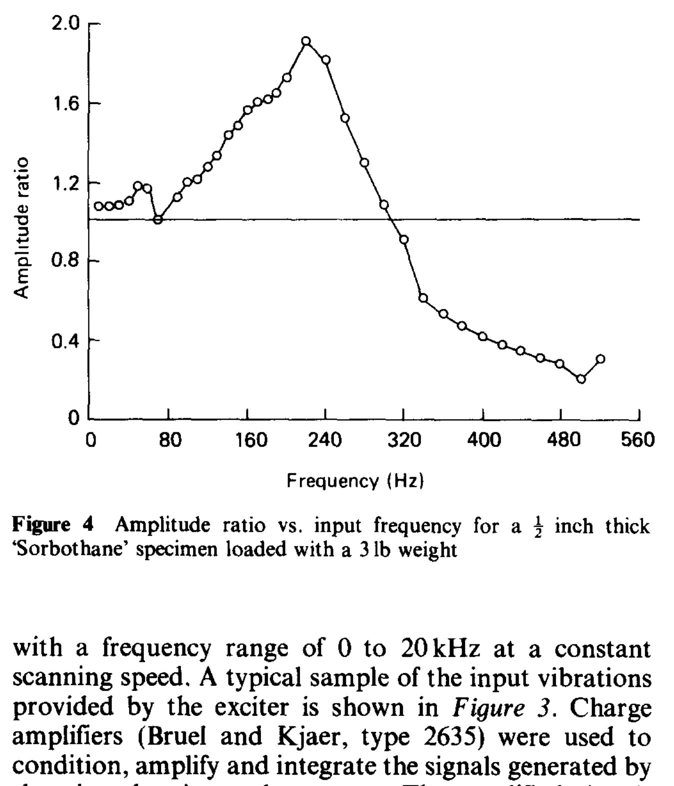

Figure 4 Amplitude ratio vs. input frequency for a $\frac{1}{2}$ inch thick 'Sorbothane' specimen loaded with a 3 lb weight

with a frequency range of 0 to 20 kHz at a constant scanning speed. A typical sample of the input vibrations provided by the exciter is shown in *Figure 3*. Charge amplifiers (Bruel and Kjaer, type 2635) were used to condition, amplify and integrate the signals generated by the piezoelectric accelerometers. The amplified signals from the accelerometers were sent to a Fourier analyser (Hewlett–Packard, 3582 A). The Fourier analyser served to sample the data and transform them into the frequency domain. The Fourier transform of each signal (at a certain frequency) was sent to an IBM PC data-acquisition system which calculated the transfer function for each frequency. After scanning the frequency range of 0 to 20 kHz, the transfer function (amplitude ratio) $H(j\omega)$ was computed and displayed by the data-acquisition system. The amplitude ratio has been used as a measure of the antivibration characteristics of the tested polymers. The lower is the amplitude ratio, the better is the shock-absorption capability of the polymer.

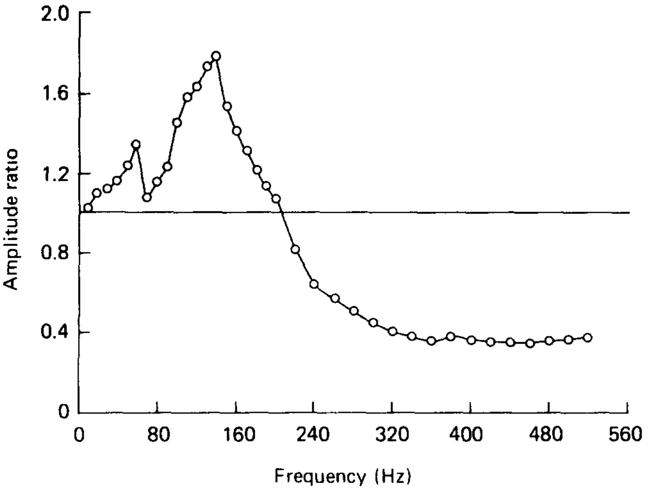

Figure 5 Amplitude ratio vs. input frequency for a $\frac{1}{2}$ inch thick 'Sorbothane' specimen loaded with a 6 lb weight

RESULTS AND DISCUSSION

The frequency-dependent antivibration characteristics of 'Sorbothane' under conditions of different loading are given in *Figures 4* and *5*. These figures show that, for a certain input frequency range, the amplitude ratio exceeds a value of unity, indicating the ineffectiveness of the polymer as a shock absorber. At higher frequencies, the amplitude ratio decreases to a fairly steady value below unity. A comparison of *Figures 4* and *5* shows that an increase in the applied load on a shock-absorbing system of the same basic characteristics reduces the frequency range over which the polymer is ineffective, with a concomitant increase in the useful frequency range of the polymer.

A comparison of the amplitude ratio vs. frequency curves obtained from model tests with various shock-absorbing polymers can help to select the best polymer for a specific application.

One advantage of this experimental technique is that the nature of the oscillatory input provided by the shaker can be varied to give different types of inputs like sinusoidal, square, ramp and random oscillations. Thus, the shock-absorption characteristics of materials can be tested under different types of vibration.

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